How the procedure will work:

The mass of an accelerating cart can be determined using only a meter stick, stop watch and a few known masses as well as a two fundamental physics equations. The first step in the process will be to calculate the acceleration of the cart by measuring the time it takes to move through a fixed distance. This is a simple calculation using the equation

\[ d = v_0 t + \frac{1}{2} at^2 \]

The cart will be starting from rest so the velocity term will go to zero and the equation can be rearranged to solve for the acceleration:

\[ a = \frac{2d}{t^2} \]

After calculating the acceleration, as shown below in table 1, the mass of the accelerating system can be found using Newton’s second law.

\[ F_{net} = ma \]

However, Newton’s second law has two restrictions. First, it can only be used on one object at a time and second, it can only be used in one dimension at a time. The lab design has TWO interacting objects which are both moving in DIFFERENT dimensions. Therefore, it will be necessary to setup the second law for each object and then algebraically combine the equations. The combination is allowed because each accelerating object shares two things which must be equal. They are acceleration and tension. The free body diagrams below show the forces acting on each object.

The vertical forces in FBD 2 can be ignored for the purpose of this lab because they do not contribute in any way to the horizontal acceleration of the cart. Applying Newton’s Second law to the hanging mass and cart separately produces the following two equations:

\[ T - m_1 g = -m_2 a \quad (1) \]
\[ T = m_2 a \quad (2) \]
The mass in equation 1 represents the mass hanging from the pulley. The right side of the equation is negative because the mass is accelerating in the downward direction. The mass in equation 2 represents the mass of the cart and all of its contents. Additionally, the tension and acceleration are both negative but the negative sign cancels from both sides of the equation. These equations can then be combined by T because it is the unknown and unusable variable.

\[-m_2a = m_1a - m_2g\]
\[m_1g = m_2a + m_1a\]
\[m_1g = (m_2 + m_1)a\]

Notice the final equation is still in the form of \(F_{\text{net}} = ma\). In this case the mass in parenthesis represents the mass of the entire system. Also notice it is an equation in the form of a straight line \((y = mx + b, \text{ where } b = 0)\). Hence, a graph can be constructed of \(F\) vs. \(a\). This should produce a straight line whose slope is equal to the mass of the entire system. The final step in finding the mass of the cart would be to subtract all other masses from the slope. As long as the mass of the entire system remains constant they can be moved around to change the size of the pulling force.

Data:

The collected data can be found in the table below:

<table>
<thead>
<tr>
<th>Pulling Force (N)</th>
<th>Time (s)</th>
<th>Accelerating distance (m) 0.60</th>
<th>(t_{\text{avg}})</th>
<th>(a \text{ (m/s}^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.49</td>
<td>1.46</td>
<td>1.31</td>
<td>1.37</td>
<td>1.38</td>
</tr>
<tr>
<td>0.686</td>
<td>1.11</td>
<td>1.15</td>
<td>1.10</td>
<td>1.12</td>
</tr>
<tr>
<td>0.882</td>
<td>1.02</td>
<td>1.00</td>
<td>1.05</td>
<td>1.02</td>
</tr>
<tr>
<td>1.078</td>
<td>1.00</td>
<td>0.98</td>
<td>0.94</td>
<td>0.97</td>
</tr>
<tr>
<td>1.274</td>
<td>0.96</td>
<td>0.88</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td>1.47</td>
<td>0.72</td>
<td>0.83</td>
<td>0.81</td>
<td>0.79</td>
</tr>
</tbody>
</table>

The following graph includes a linear regression line with equation and an \(R^2\) value to indicate the quality of fit.
Calculation for mass of cart:
Because the slope of the best fit line is equal to the mass of the entire system, the mass of the cart can be found by subtracting all other system masses from this value.

\[ M = m_{\text{cart}} + m_{\text{hanger}} + m_{\text{in cart}} \]
\[ m_{\text{cart}} = M - m_{\text{hanger}} - m_{\text{in cart}} \]
\[ m_{\text{cart}} = 0.804 - 0.15 - 0.500 \]
\[ m_{\text{cart}} = 0.154 \text{kg} \]

Analysis
According to calculations, the mass of the cart is almost exactly 100 grams under mass since the given value of the cart is 0.255 kg. Such an even number opens up the possibility there was an error in properly recording the masses used in this lab. Nonetheless, other factors may be contributing to this discrepancy. For example, the R^2 value is very close to 1 which indicates the data fits the best-fit line fairly well; however, by observation it appears the last data point (1.47, 1.94) could be an outlier considering the short time interval which is to be measured. The effect of this data point on the line would be to decrease the slope which in turn decreases the calculated mass of the cart. If this data point is omitted, the calculated mass of the cart becomes 339 grams which is closer to the accepted value but not by much (see Appendix A).

There is also the possibility reaction time played a significant role in the discrepancy of the lab because of the short time intervals involved. For example, the last data point which is thought to be discrepant has an average time of 0.79 seconds. Normal reaction times range from about 0.15 seconds to 0.25 seconds in the worst cases. The first and last data points can be adjusted for maximum and minimum values using an average reaction time of 0.20 seconds. These points can then be plotted in the worst case scenarios to produce a minimum and maximum possible slope. Reaction time alone cannot account for the missing mass if the true mass of the cart does not fall within this range. The graph below shows the results of using minimum and maximum slope.

![Graph showing F vs a with equations and R^2 values: y = 2.6427x - 1.7876, y = 0.7487x + 0.0182, y = 0.3734x + 0.1682.](image)

The equations in the graph above show the slopes for the lines in the order of maximum to minimum. These slopes correspond to masses of the cart ranging from 1.95 kg to a -0.277 kg. Given the wide range of
possibilities using absolute minimums and maximums, it is likely the reaction time in this lab is far better than 0.20 seconds.

**Conclusion**

The mass of the cart could not be calculated with any degree of precision as indicated by the graph of absolute uncertainty. These are absolute values and the true mass of the cart falls within that range. However, the absolute ratings are just that. They represent the WORST POSSIBLE combinations which is statistically unlikely when uncertainty is randomly distributed throughout data. Therefore, the mass of the cart as calculated is probably representative of what the collected data should show. At this time it is not possible to say if the lab could be considered a success. It should be repeated using photogate timers or some other method which would significantly decrease the uncertainty of the time measurement. A proper analysis at that time will reveal if there are problems associated with the procedure which need to be corrected.
**Appendix A**

<table>
<thead>
<tr>
<th>Pulling Force (N)</th>
<th>Time (s)</th>
<th>tavg</th>
<th>a (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trial 1</td>
<td>Trial 2</td>
<td>Trial 3</td>
</tr>
<tr>
<td>0.49</td>
<td>1.46</td>
<td>1.31</td>
<td>1.37</td>
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<td>0.72</td>
<td>0.83</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Slope of line 0.99
Mass of cart 0.339 kg

Table 1

![Graph 1](image)

The above graph shows the effect of eliminating the final data point from the calculation. Not only is there the desired mass increase, but the R² value increased as well. While this did produce a more attractive result, there are no sufficient reasons for the exclusion of the data point from the analysis.

**Appendix B**

*Note to students:* a more advanced statistical method was used at the conclusion of my analysis to determine a more reasonable uncertainty for the calculated mass of the cart. In this experiment, reaction time alone was NOT able to account for the discrepancy between the known mass and calculated mass. The more reasonable uncertainty produces a range of masses of 129 grams \( \rightarrow \) 180 grams. Simply, this means time alone cannot account for the discrepancy and there must be some other problem in the data collection or lab procedure.

*Additional Note:* I used raw student data for this example. As such I cannot guarantee the accuracy of all comments as they apply to this specific lab design for which the data was collected, but they should be fairly representative of the average.